$(a+b)^a$, Cumulative Credibility, and the Distribution Envelope Determination (DEnv) Algorithm

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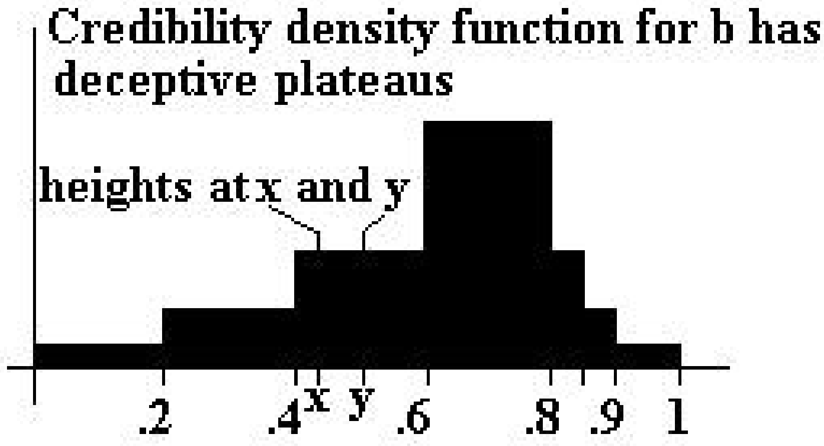
Problem 2a



Evidence for x = Evidence for y

But -

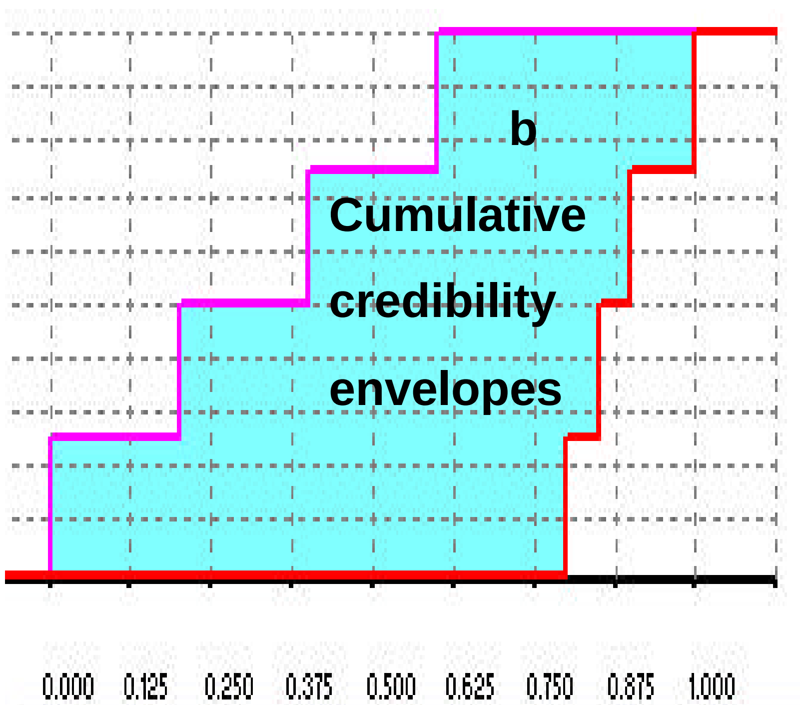
There is no evidence they have equal credibility

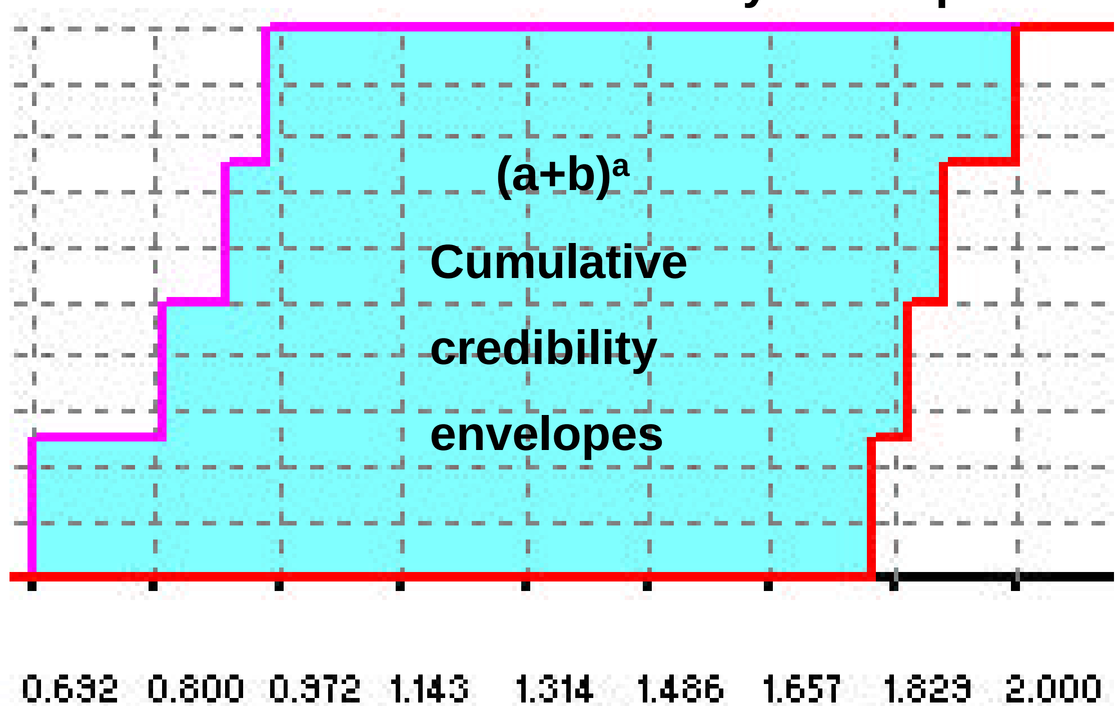
Distribution of credibility over its interval could be highly skewed, uniform, an impulse, etc.

Total credibility is the same for each interval

This uncertainty about the density function is easily expressed as:

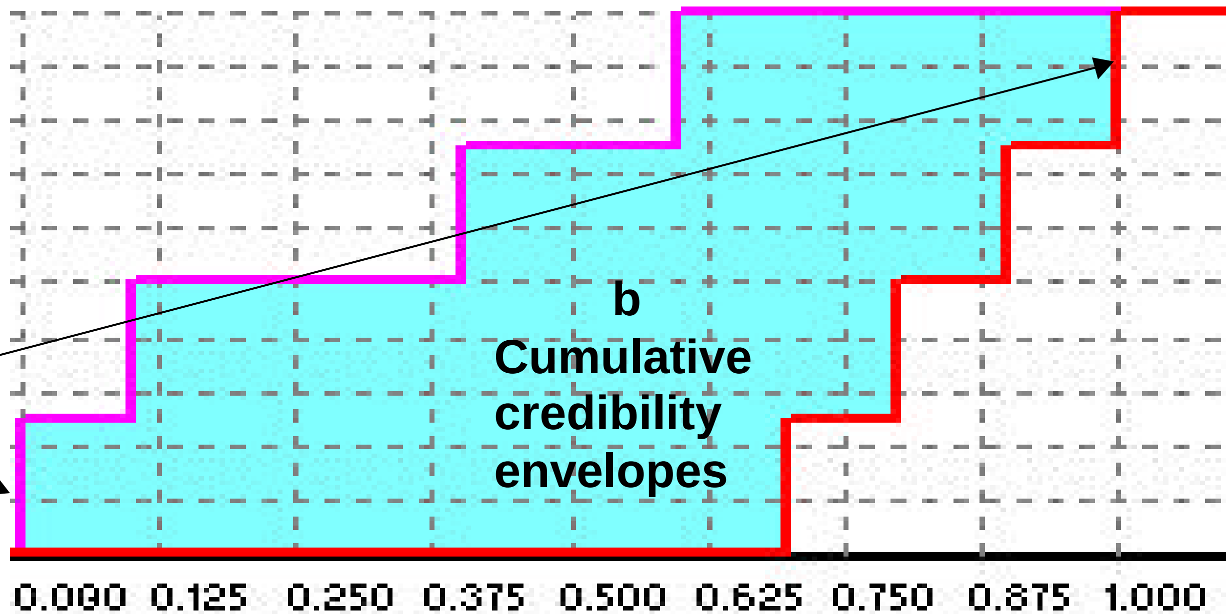
Cumulative credibility envelopes



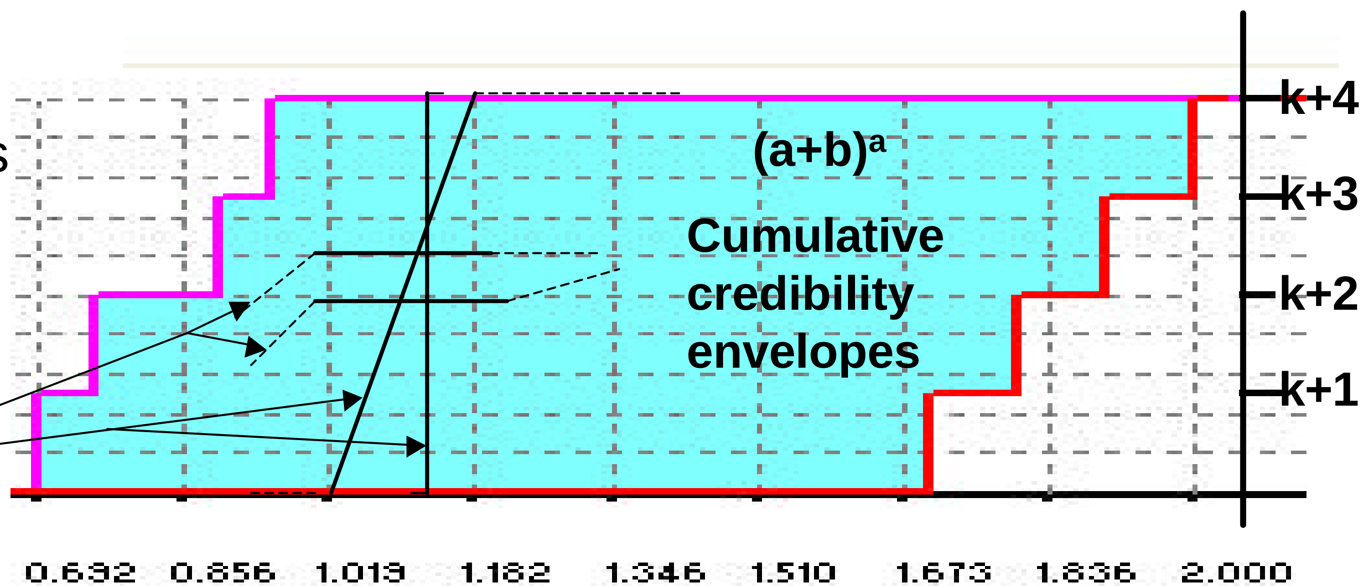


Problem 2b: Interpreting Envelopes

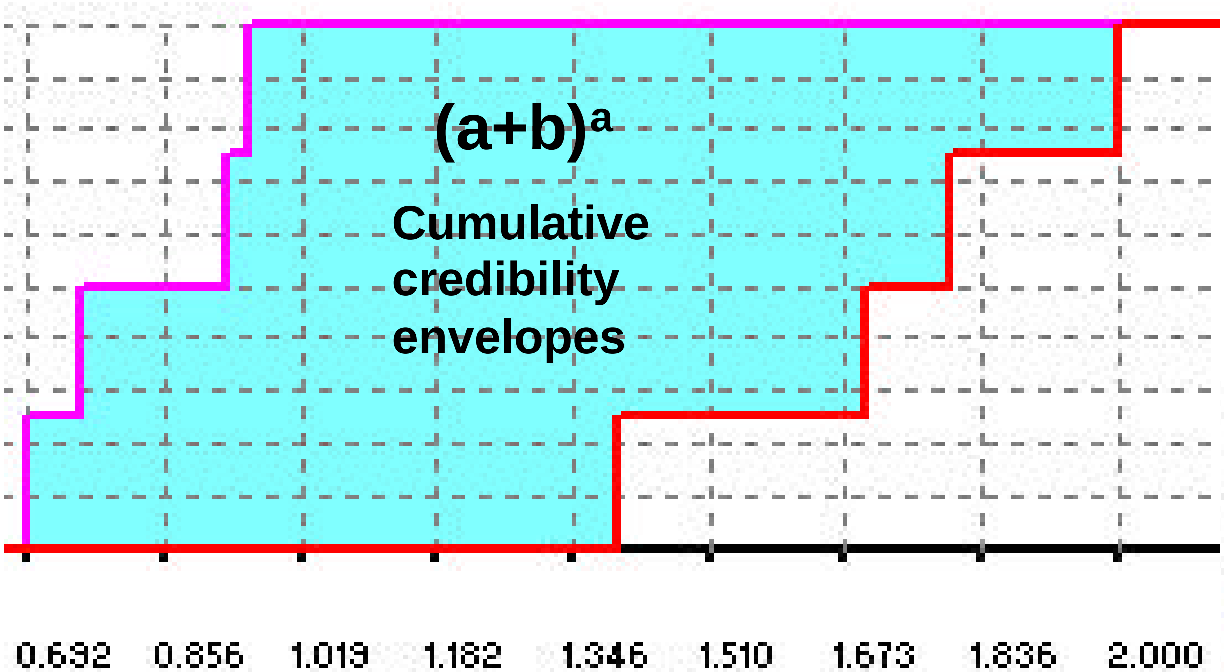
1 unit of credibility distributed over [0,1] in an undefined manner implies cumulative credibility can increase as early as
1 unit at b=0
and as late as
1 unit at b=1.
Similarly for other intervals.



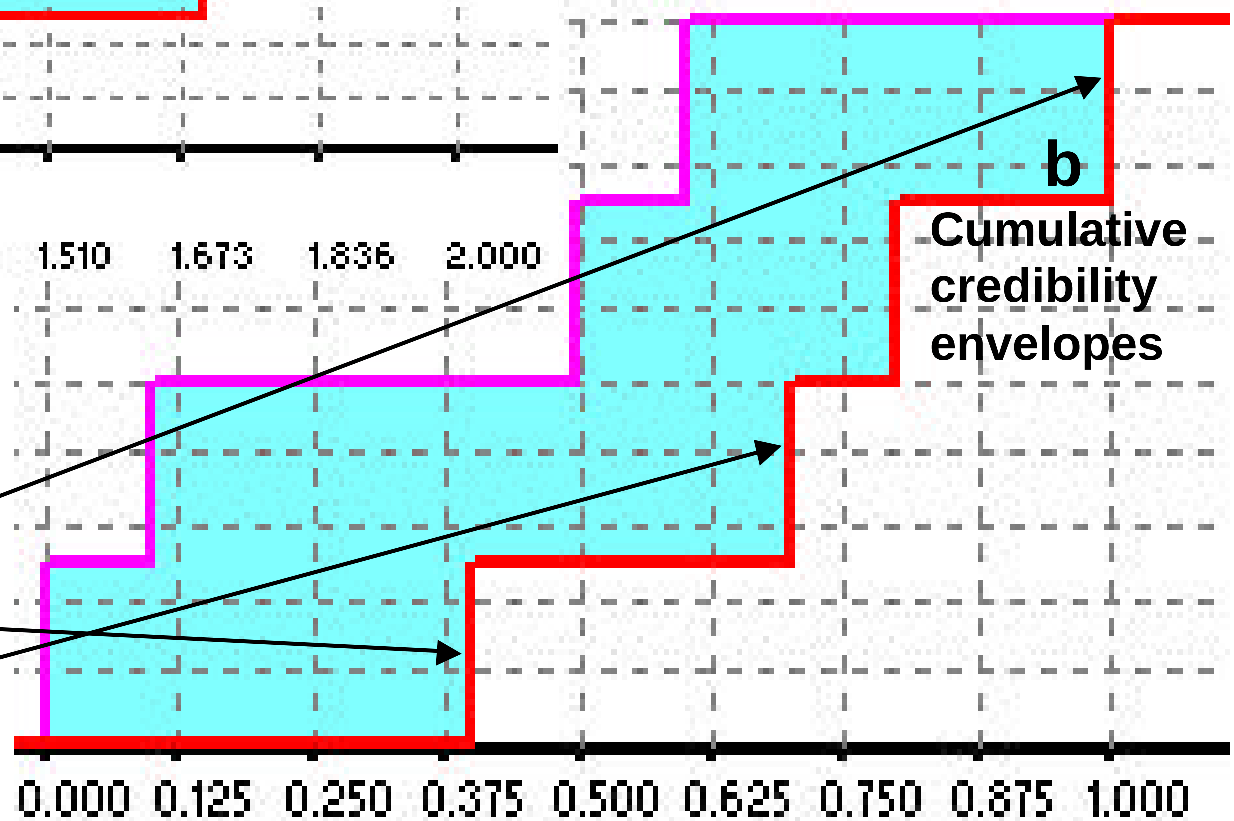
Any cumulation curve within the envelopes is plausible. So, over [1,1.2]
credibility may increase as little as 0,
or as much as 4.



Problem 2c



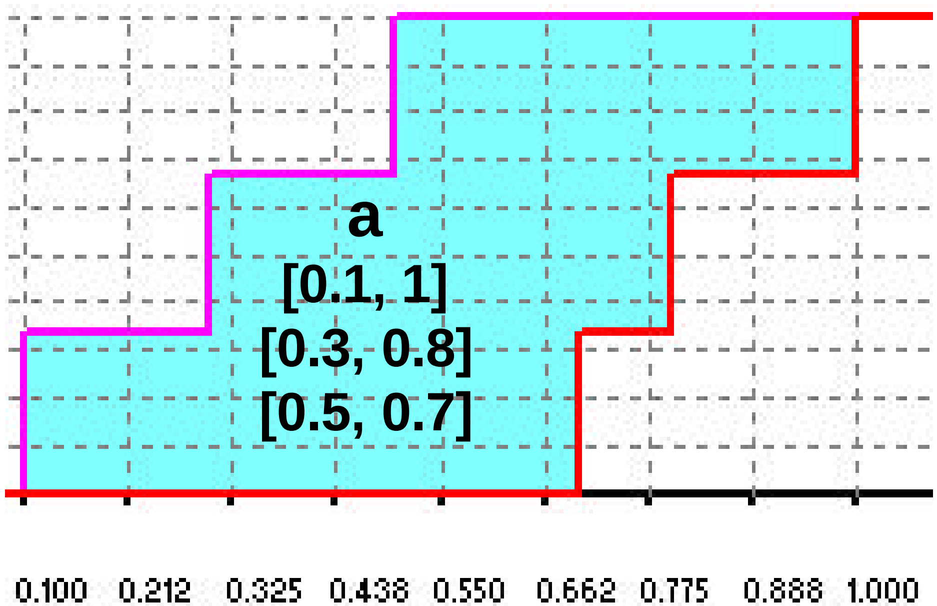
Not shown: credibility of values below (above) the lowest (highest) measurement bounds

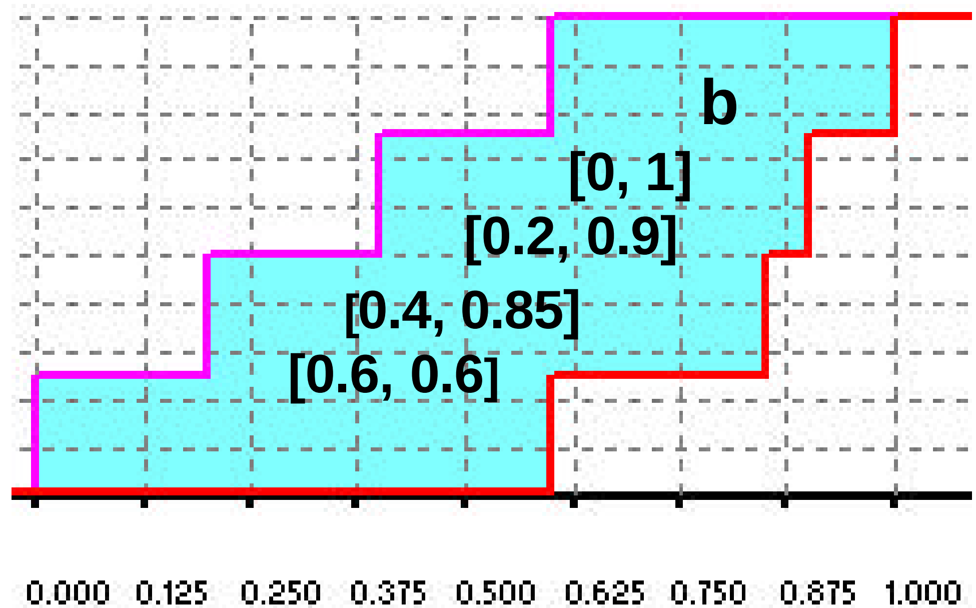


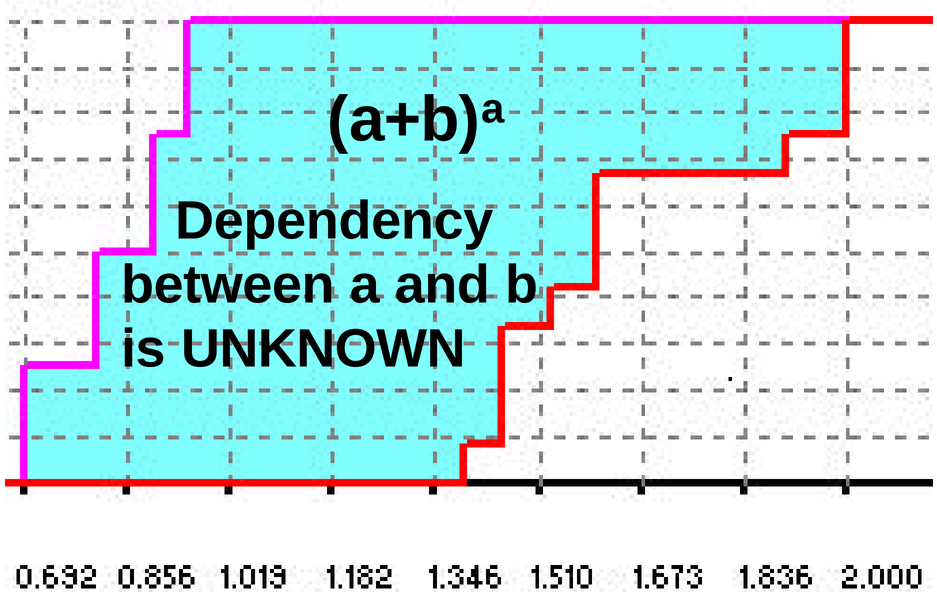
b is described with these intervals:

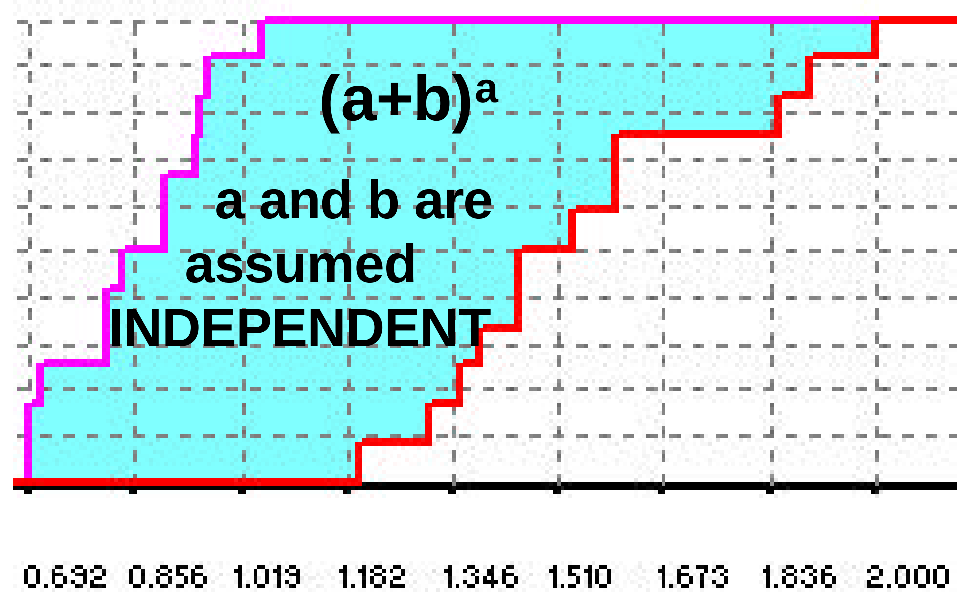
[0,1]
[0.1,0.4]
[0.5,0.7]
[0.6,0.8]

Problem 3a


a
[0.1, 1]
[0.3, 0.8]
[0.5, 0.7]
0.100 0.212 0.325 0.438 0.550 0.662 0.775 0.888 1.000


b
[0, 1]
[0.2, 0.9]
[0.4, 0.85]
[0.6, 0.6]
0.000 0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000


(a+b)a
Dependency
between a and b
is UNKNOWN
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000


(a+b)a
a and b are
assumed
INDEPENDENT
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000

The DEnv Algorithm –
a Independent of b

- Given intervals for **a** and **b**
  - Make a *joint distribution tableau*

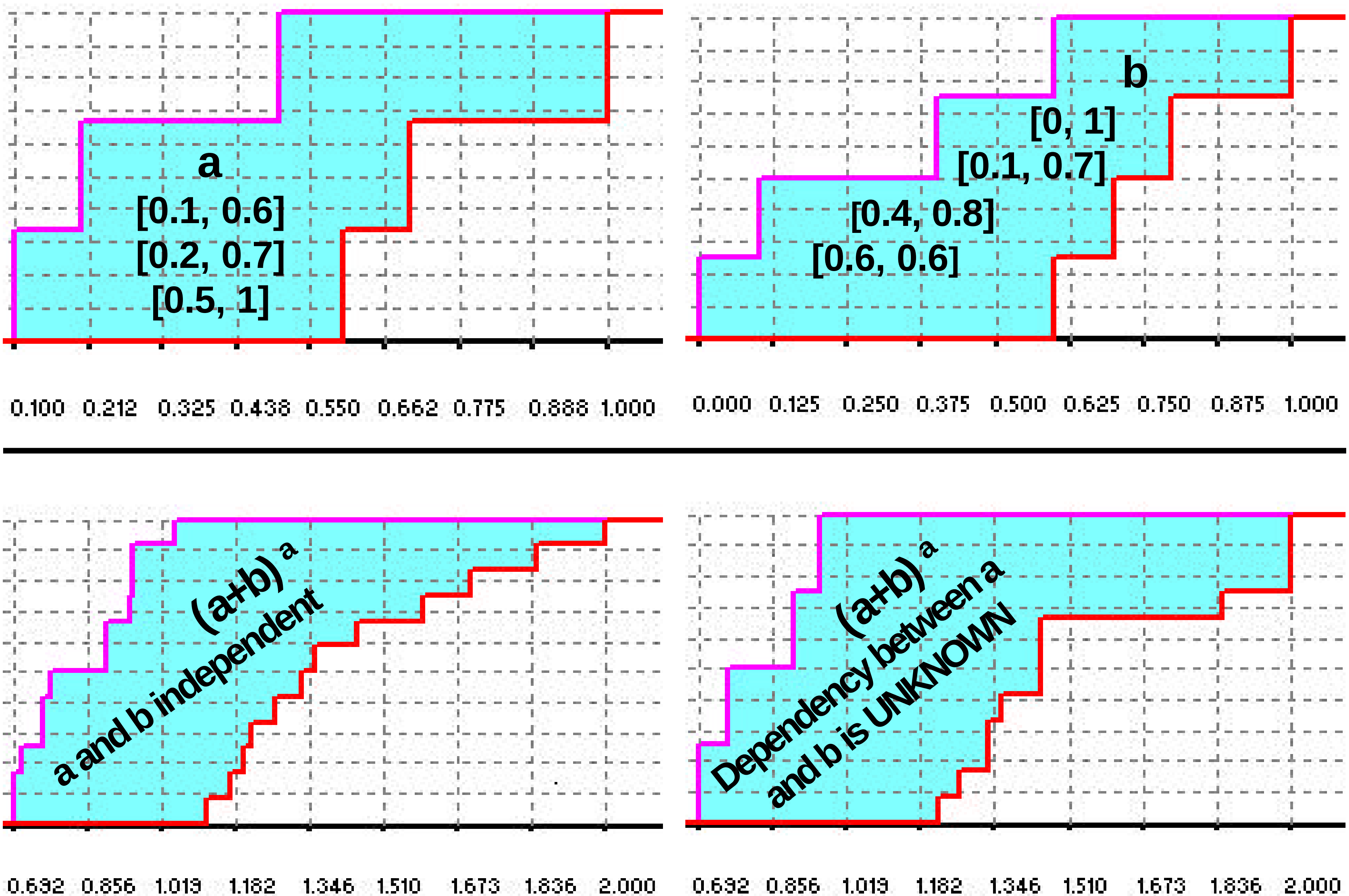
b? $(a+b)^a$?a ↘	[0, 1]	[.2,.9]	[.4,.85]	[.6,.6]
[.1,1]	[.69,2]	[.81,1.9]	[.9,1.85]	[.96,1.6]
[.3,.8]	[.69,1.6]	[.81,1.5]	[.9,1.49]	[.97,1.31]
[.5,.7]	[.71,1.7]	[.84,1.39]	[.95,1.36]	[1.05,1.2]

- Independence implies equal interior cell credibilities
- If **a** or **b** is one interval, there is one row or column
- Left (right) envelope height at any value z is the cumulated credibilities of interior cells with low (high) bounds <= (>=) z

The DEnv Algorithm – Unknown Dependency Between a and b

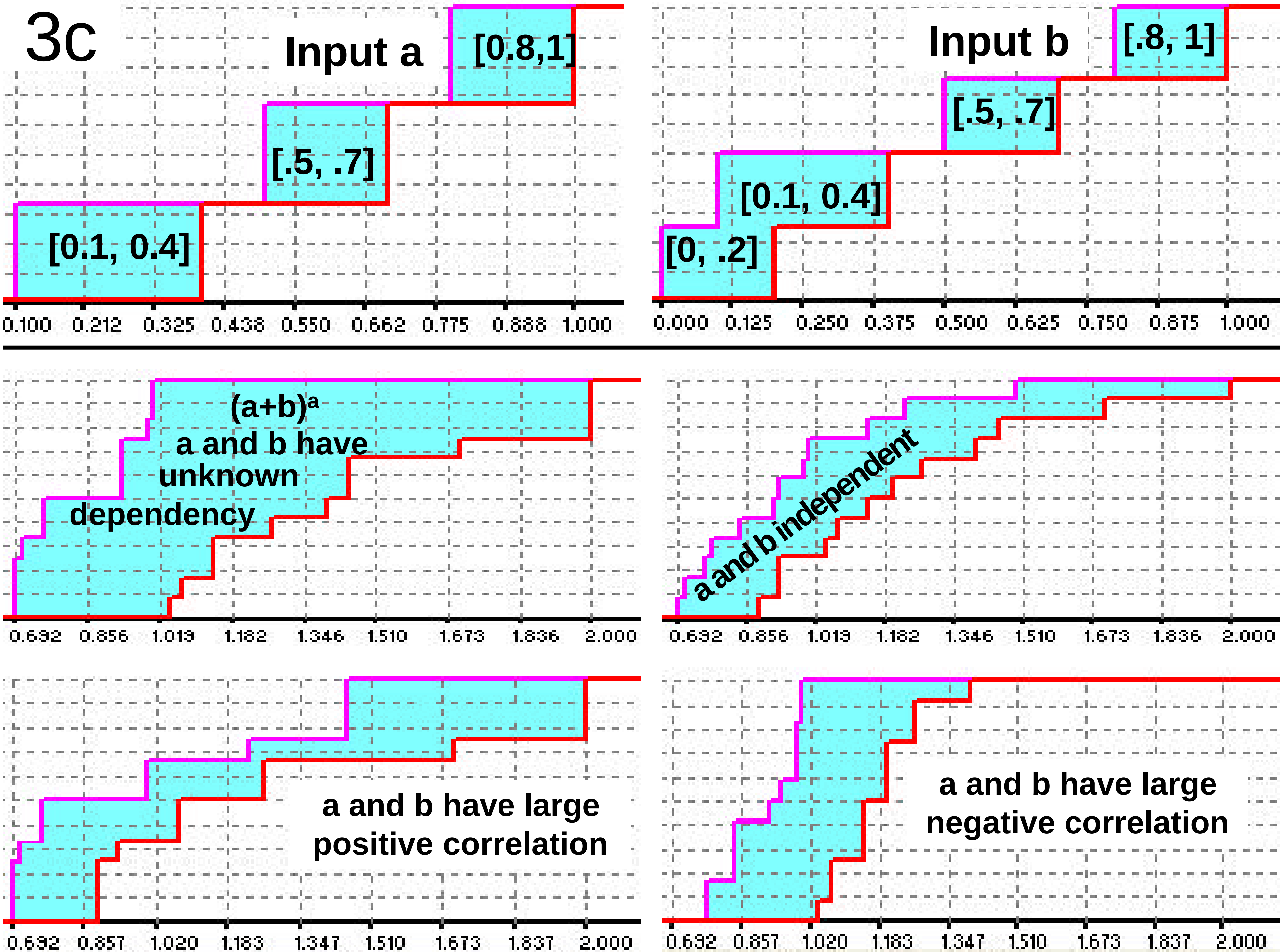
- Credibilities of interior cells need not be equal
- Credibilities of interior cells in a row sum to credibility of marginal cell
  - Analogously for columns
- Within those constraints, maximize (minimize) cumulation at each z to get left (right) envelope
  - Use linear programming
- (Caveat: credibility calculus is currently incomplete)
  - (Sub-caveat: Statool graphs treat credibility as probability)

Problem 3b



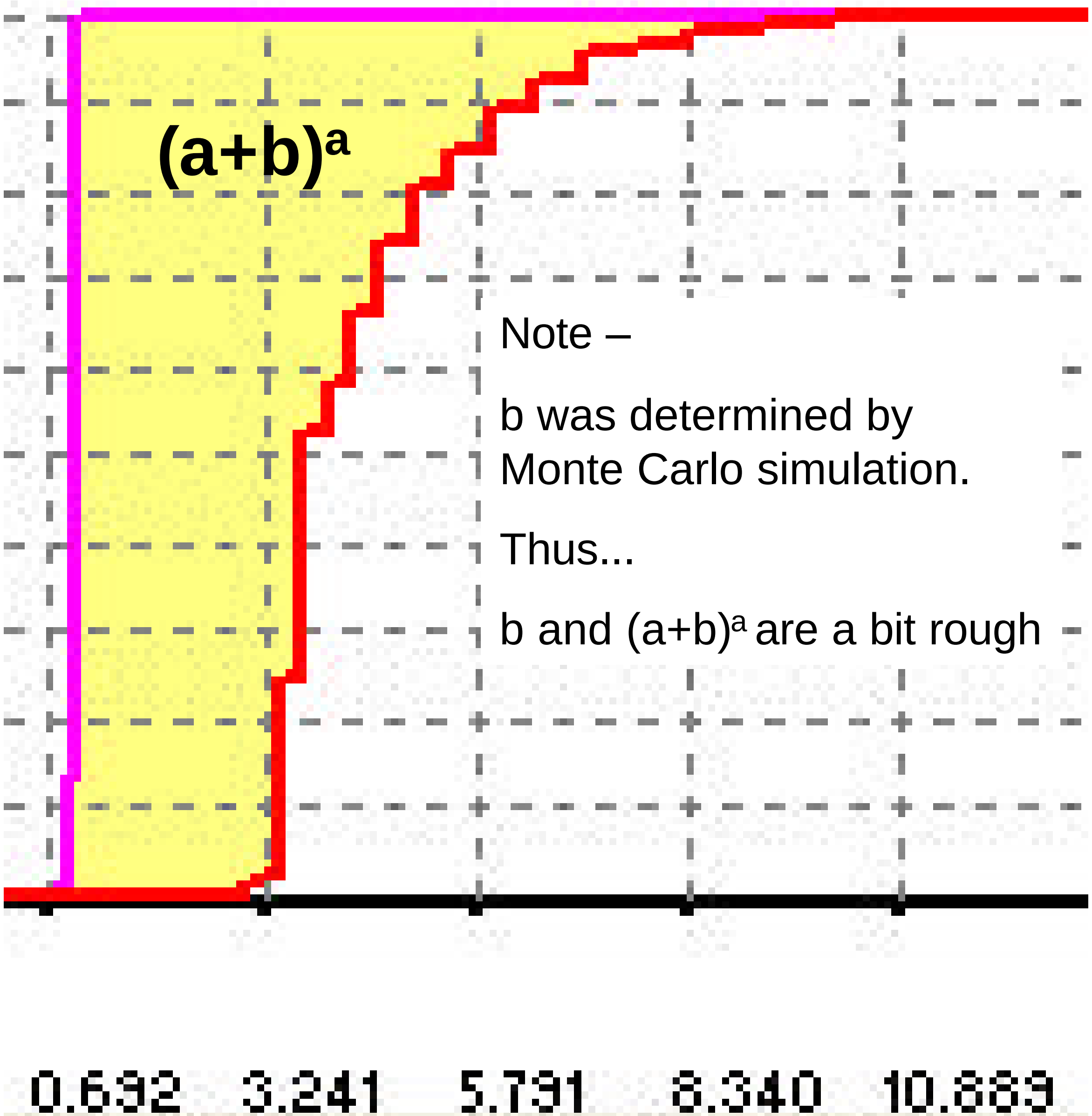
a
[0.1, 0.6]
[0.2, 0.7]
[0.5, 1]
0.100 0.212 0.325 0.438 0.550 0.662 0.775 0.888 1.000
b
[0, 1]
[0.1, 0.7]
[0.4, 0.8]
[0.6, 0.6]
0.000 0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000
(a+b) a
a and b independent
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000
(a+b) a
Dependency between a and b is UNKNOWN
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000

3c



Input a
[0.1, 0.4]
[.5, .7]
[0.8,1]
0.100 0.212 0.325 0.438 0.550 0.662 0.775 0.888 1.000
Input b
[0, .2]
[0.1, 0.4]
[.5, .7]
[.8, 1]
0.000 0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000
(a+b)a
a and b have
unknown
dependency
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000
a and b independent
0.692 0.856 1.019 1.182 1.346 1.510 1.673 1.836 2.000
a and b have large
positive correlation
0.692 0.857 1.020 1.183 1.347 1.510 1.673 1.837 2.000
a and b have large
negative correlation
0.692 0.857 1.020 1.183 1.347 1.510 1.673 1.837 2.000

Problem 4 – a an Interval, b Envelopes



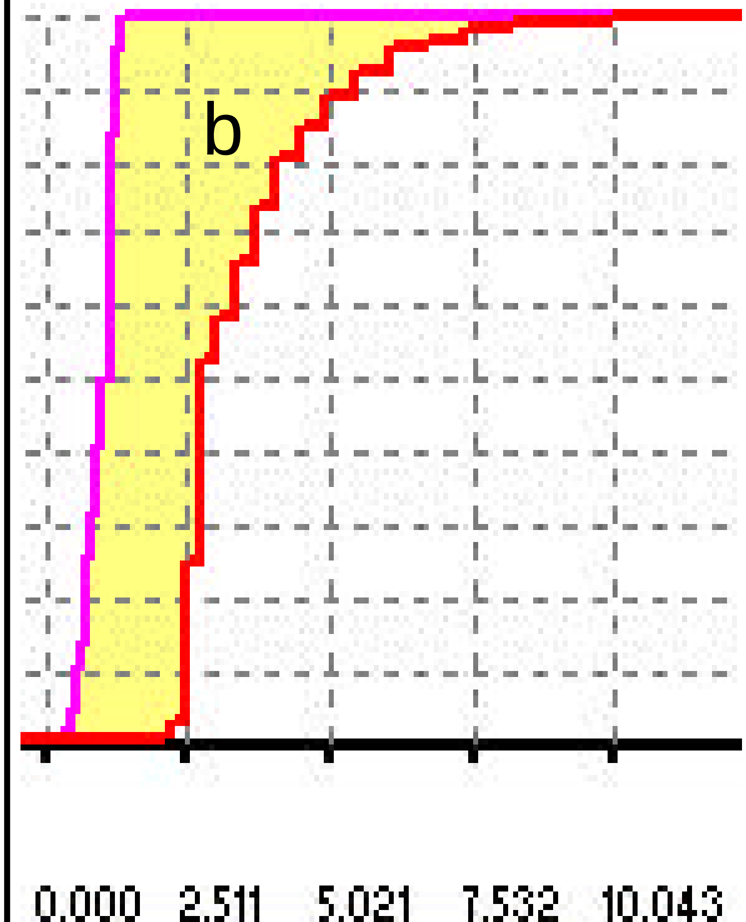
Note –

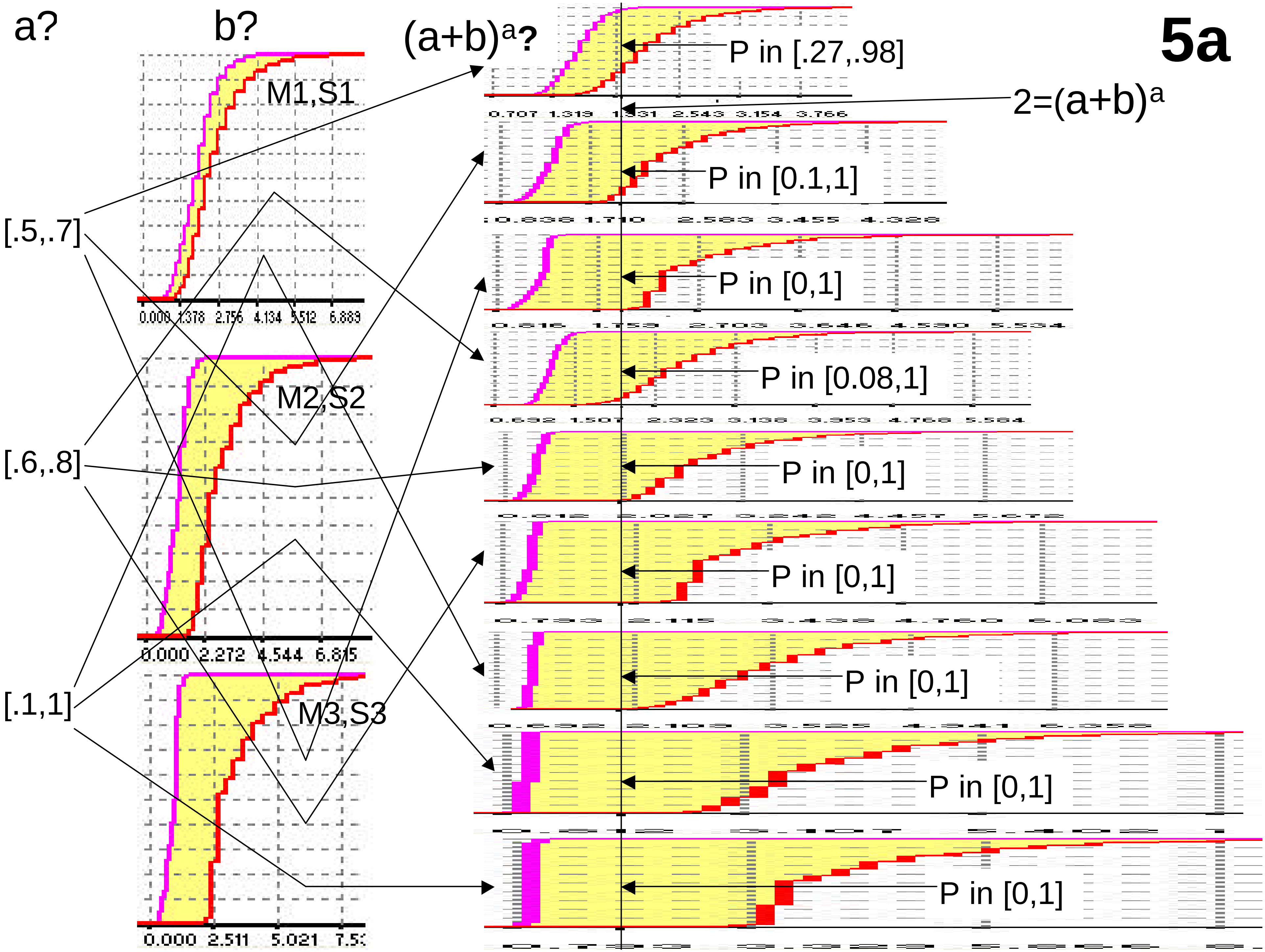
b was determined by Monte Carlo simulation.

Thus...

b and $(a+b)^a$ are a bit rough

a=[0.1, 1]



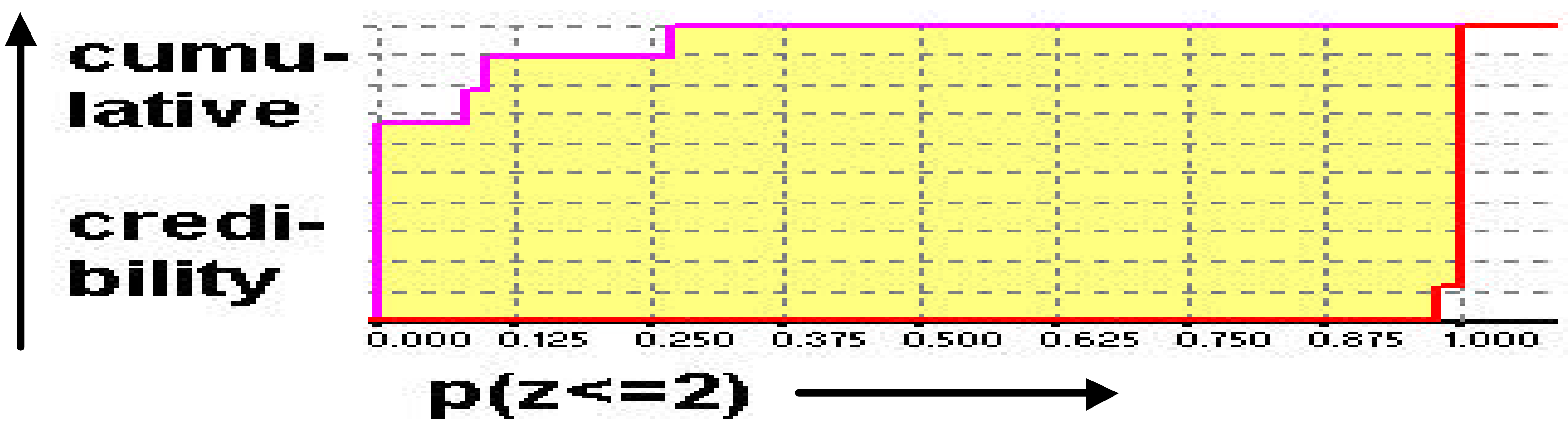


5a
a?
b?
(a+b)a?
M1,S1
M2,S2
M3,S3
[.5,.7]
[.6,.8]
[.1,1]
P in [.27,.98]
2=(a+b)a
P in [0.1,1]
P in [0,1]
P in [0.08,1]
P in [0,1]
P in [0,1]
P in [0,1]
P in [0,1]
P in [0,1]
0.000 1.378 2.756 4.134 5.512 6.889
0.000 2.272 4.544 6.815
0.000 2.511 5.021

Problem 5a (cont.)

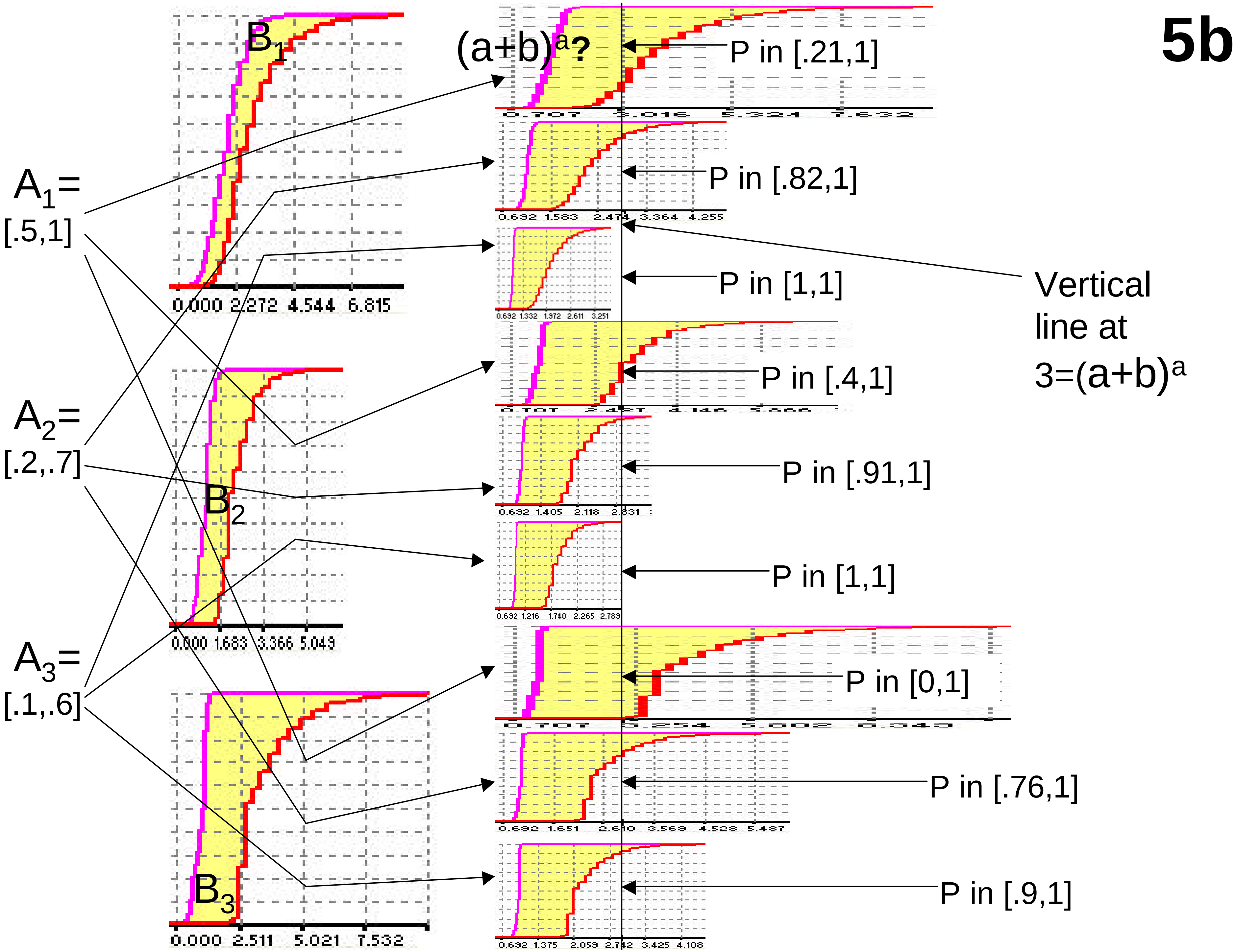
- Nine equally credible envelopes for z
  - (Assuming measurements of **a** are independent of the credibility of measurements of **b**)
  - (Otherwise envelopes are not equally credible)
- Thus p(z=2) has 9 equally credible intervals:
  - [.27,.98], [.1,1], [0,1], [.08,1], [0,1], [0,1], [0,1], [0,1], [0,1]
  - This enables…

Envelopes around the cumulative credibility of p(z=2):



5a – Further Comments

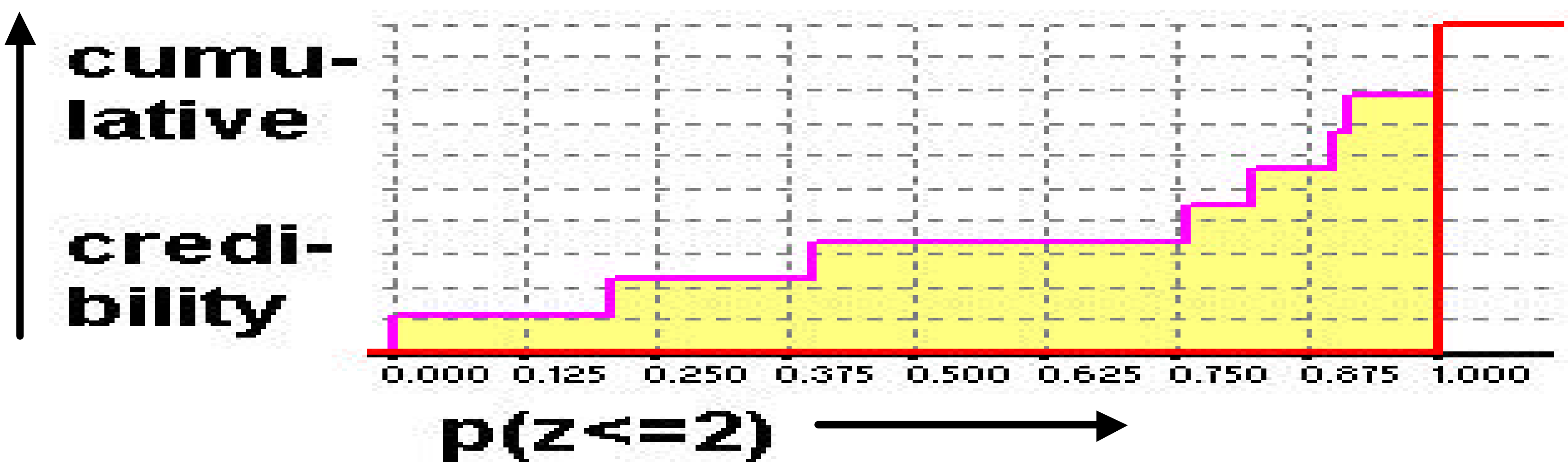
- If credibilities of measurements of **a**
  - are not independent of credibilities of **b**
  - Then envelope step heights can differ
- Concept of credibility must compensate for
  - useless certainty
  - probability in [0,1] is certain but useless
    - its credibility should affect other measurements in a reasonable way

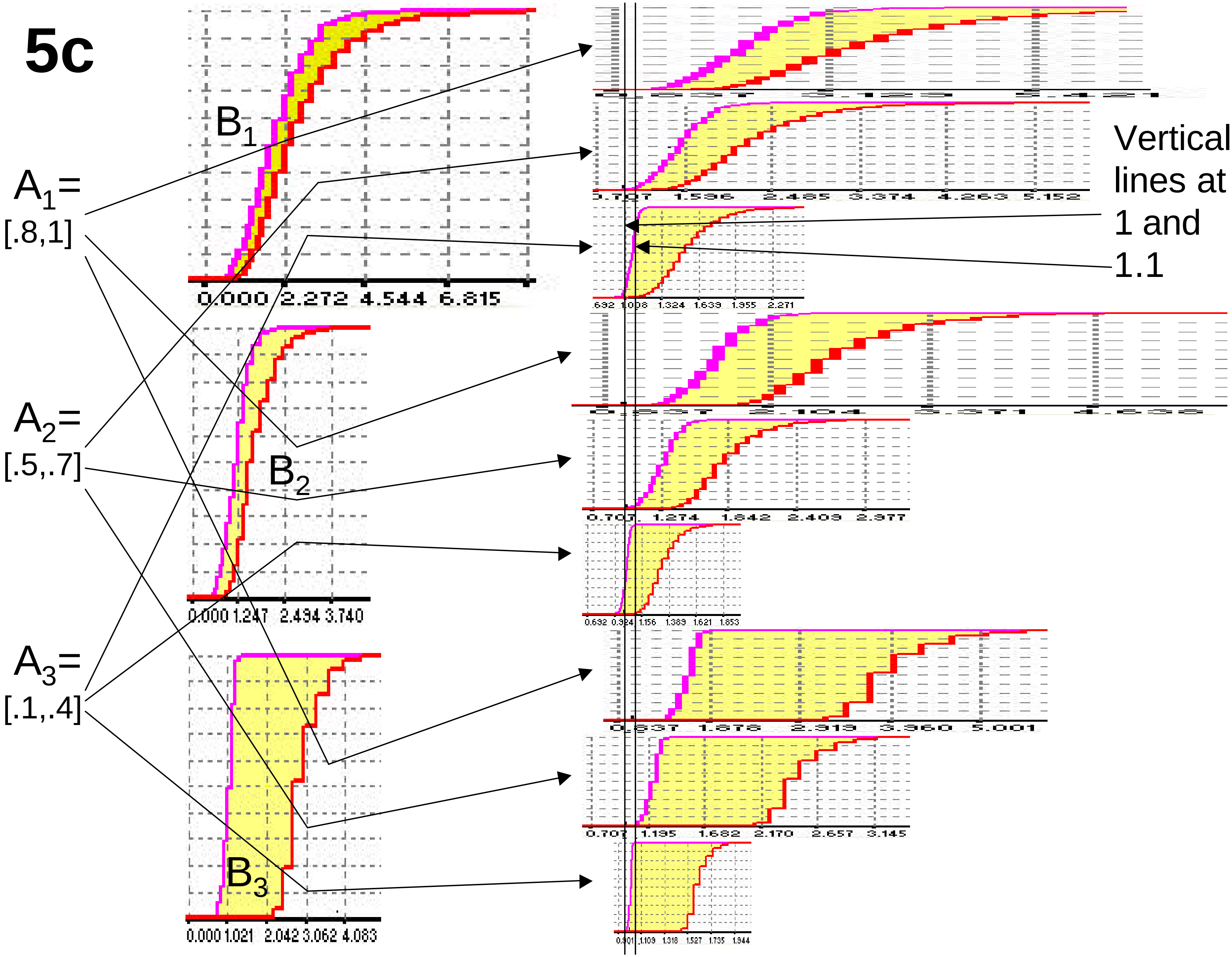

5b
(a+b)a?
A1=
[.5,1]
A2=
[.2,.7]
A3=
[.1,.6]
B1
B2
B3
0.000 2.272 4.544 6.815
0.000 1.683 3.366 5.049
0.000 2.511 5.021 7.532
P in [.21,1]
P in [.82,1]
P in [1,1]
P in [.4,1]
P in [.91,1]
P in [1,1]
P in [0,1]
P in [.76,1]
P in [.9,1]
Vertical
line at
3=(a+b)a
0.692 1.583 2.474 3.364 4.255
0.692 1.405 2.118 2.831
0.692 1.651 2.610 3.569 4.528 5.487
0.692 1.375 2.059 2.742 3.425 4.108

Problem 5b (cont.)

- Nine equally credible envelopes for z
  - (Assuming measurements of **a** are independent of the credibility of measurements of **b**)
  - (Otherwise envelopes are not equally credible)
- Thus p(z=3) has 9 equally credible intervals:
  - [.21,1], [.82,1], [1,1], [.4,1], [.91,1], [1,1], [0,1], [.76,1], [9,1]
  - Of course, we could instead find p(z=k) for "any" k

Envelopes around the cumulative credibility of p(z=3):

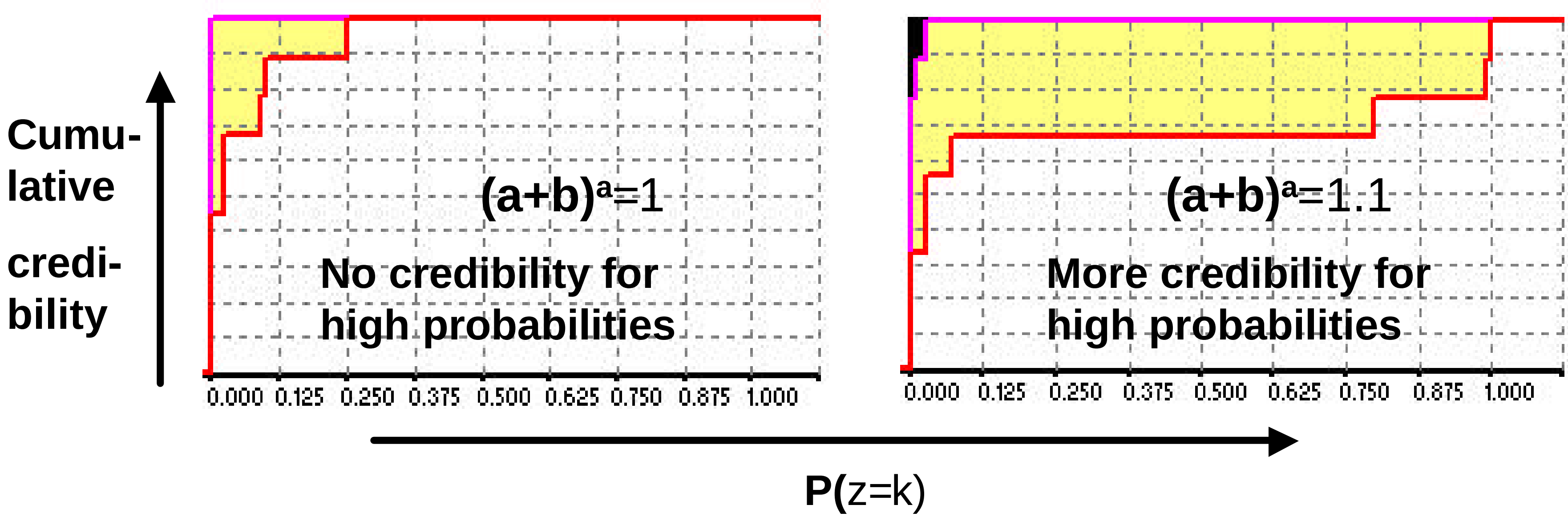




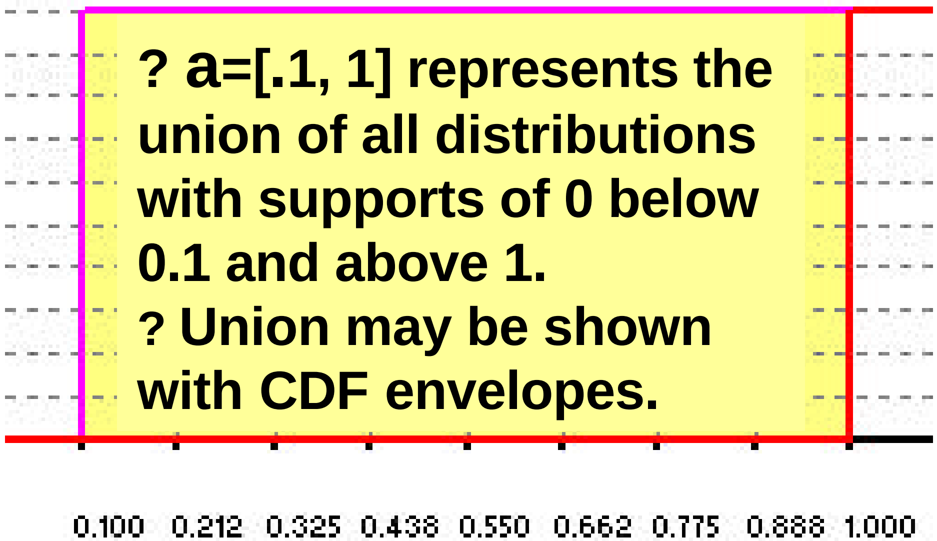
5c
B1
A1=
[.8,1]
A2=
[.5,.7]
B2
A3=
[.1,.4]
B3
0.000 2.272 4.544 6.815
0.000 1.247 2.494 3.740
0.000 1.021 2.042 3.062 4.083
Vertical lines at 1 and 1.1
1.596 2.485 3.374 4.263 5.152
1.008 1.324 1.639 1.955 2.271
0.707 1.274 1.842 2.409 2.977
0.692 0.924 1.156 1.389 1.621 1.853
1.878 2.919 3.960 5.001
0.707 1.195 1.682 2.170 2.657 3.145
1.109 1.318 1.527 1.735 1.944

Problem 5c (cont.)

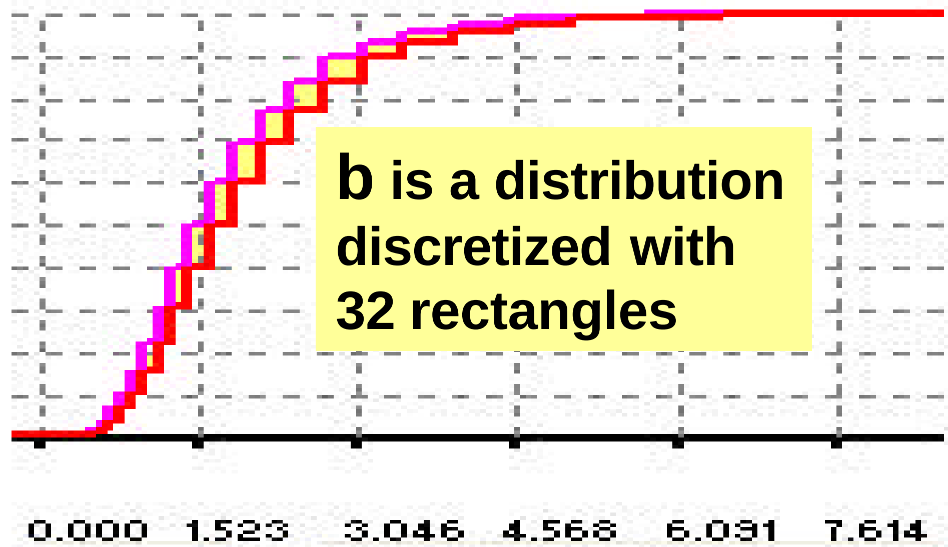
- As before, 9 envelopes for z
- Thus p(z=1) has 9 intervals:
  – (Caveat: these were determined visually from the 9 graphs)
  – [0,0], [0,.02], [0,.09], [0,0], [0,.02], [0,.25], [0,0], [0,0], [0,.1]
- Similarly p(z=1.1) has 9 intervals also:
  – [0,0], [0,.025], [.01,.8], [0,0], [0,.07], [.025,.99], [0,0], [0,.025], [0,1]



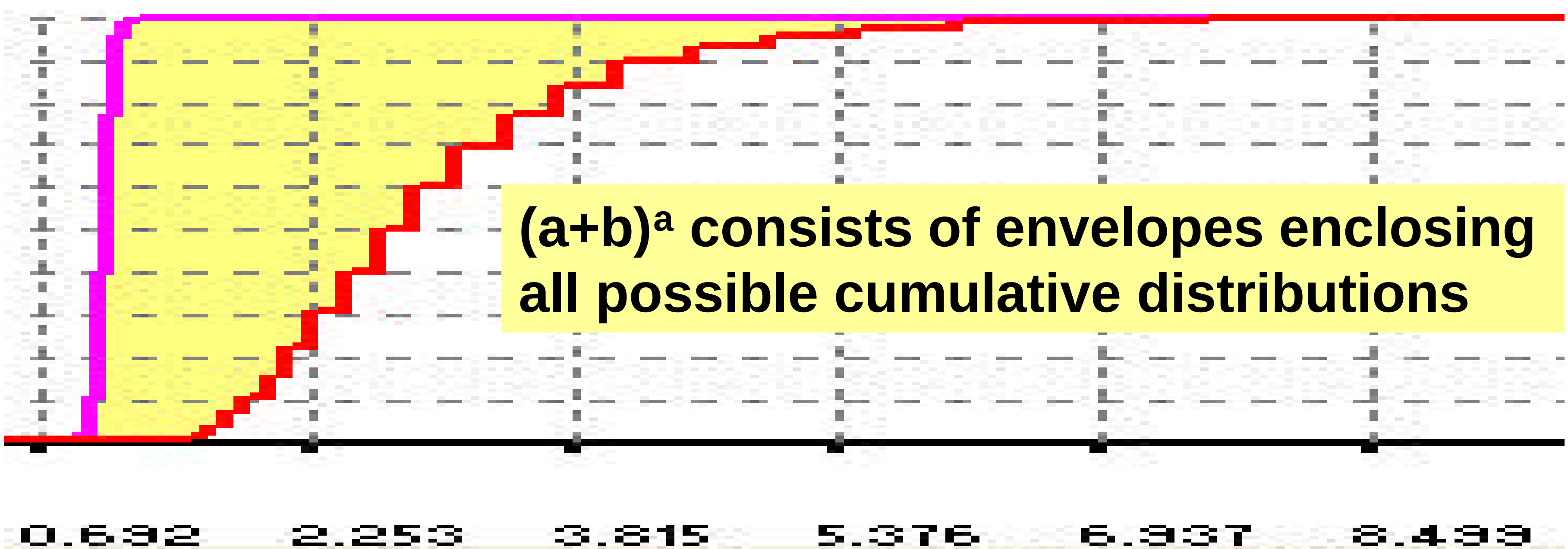
Problem 6



? a=[.1, 1] represents the union of all distributions with supports of 0 below 0.1 and above 1.
? Union may be shown with CDF envelopes.
0.100 0.212 0.325 0.438 0.550 0.662 0.775 0.888 1.000



b is a distribution discretized with 32 rectangles
0.000 1.523 3.046 4.568 6.091 7.614



(a+b)^{a} consists of envelopes enclosing all possible cumulative distributions
0.692 2.253 3.815 5.376 6.937 8.499

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